THIRD EDITION

# INTRODUCTORY METHODS OF NUMERICAL ANALYSIS

S.S. SASTRY

PHI

# Introductory Methods of Numerical Analysis

THIRD EDITION

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## Chapter 1

1. 48.21, 2.37, 52.28, 2.38, 2.38, 81.26
2. 38.46, 0.7003, 0.002222, 19.24, 2.364
3. 0.04963
5. (a) 600.05, (b) 1000.05, (c) $5.766 \times 10^{12}$
8. $348.7 \pm 0.15$
9. 701
10. $q = 3.44$
    $E_R = 0.0039$
12. $n = 9$
13. $n = 10^8 + 1$
14. $\ln 1.2 = 0.1823215$, $n = 9$

## Chapter 2

1. $-2.105$
2. 2.706
3. 1.796
4. 2.621
5. 1.466
6. 0.755
7. 2.279
8. 1.325
9. 0.657
10. 0.682
16. 0.6071
17. 0.4655
18. 1.516
19. 1.088
20. 0.0913
21. 1.068
22. 1.404
23. 0.5314
24. 1.466
25. 0.1514
26. $-0.70346$
28. 0.511
29. 0.657
30. 2.908
31. $-2.533$
32. 1.171
33. 0.739
34. 1.896
35. 1.756
36. 1.414214
37. 4.4934
38. 1.8438
40. The eight convergents are:
    1.0, 0.5, 0.66666, 0.75000, 0.666666, 0.666666, 0.69230769 and 0.68421052
42. 1.3247
43. 1.8393
44. $x^2 + 1$
47. 5.12487, 1.63668, 0.23845
51. 3.584428, $-1.848127$
53. $-1.9266$, $-1.8533$

## Chapter 3

2. 239, 371
3. Error in the tabular value for $x = 4.0$; true value is 5.75.
5. 52
6. 3, 111, $x^2 + x + 1$
7. 1.6751, 1.7081
8. 154, $\frac{1}{6}(2x^3 + 3x^2 + 13x)$
9. 257.259, 288.805
12. 96.66 thousands
13. 31
14. 0.783172
16. 1.685750
18. 6.7531
19. 13.062
20. 0.046
21. 3.503
22. 0.3165
23. 111.874930
24. 0.19573
25. $x^3 - 3x^2 + 5x - 6$
26. 12.45
30. $-\frac{1}{15}x^3 - \frac{3}{20}x^2 + \frac{241}{60}x - 3.9$
31. $f(6) = 5.66$, $x = 4.5$
32. 3250.88
34. $\frac{1}{2(x-1)} + \frac{1}{x-2} - \frac{1}{2(x+1)}$
39. 53
41. $x = 2.3124$
42. 0.0000125
43. $6.42 \times 10^{-8}$
48. 2.3124
49. 1.16314

## Chapter 4

1. $a_0 = 2.016$, $a_1 = 0.503$
2. $a_0 = 0.8$, $a_1 = 2.0$
4. $a = 2$, $b = 3$
5. $a = 1$, $b = -3$, $c = 2$
6. $a_0 = 600$, $a_1 = 0.013$
8. $a_0 = 1.04$, $a_1 = -0.2$, $a_2 = 0.24$
9. $a = 2$, $b = 3$
10. $a \approx 78$, $b \approx -0.8$
11. $a = 0.1$, $b = 3.0$
12. $c = 0.56$, $b = 1.05$

## Chapter 5

1. $-0.05$
2. 4.054
3. 9.66
4. 3.8140
5. (i) 0.3950, (ii) 0.3341, (iii) 0.2719

$$\left[\frac{d^2y}{dx^2}\right]_{x=3} = -0.0256$$

7. $y'(2) = 29.0$
9. 66.33
10. 1.1000, error = 0.0014 too large,
    1.0987, error = 0.0001



11. (a) 3.14, (b) $-0.747$
12. 177.4830
13. $3\frac{5}{9}$ km
14. 0.77500, 0.78279, 0.78475, 0.7854
15. 29.0993
16. 1.1873
17. 0.83865
18. 0.02020, 0.000173
24. 0.7854
25. 0.004999
26. 1.5
27. 1.00002
29. $-0.19762761$
31. 0.643269, 0.373150, 0.269583
33. 1.1454

## Chapter 6

4. $\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

5. (a) $\frac{1}{4}\begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -8 & 10 \\ 0 & 2 & -3 \\ 0 & -1 & 2 \end{bmatrix}$

6. $A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix}$, $x = \frac{9}{4}$, $y = -\frac{9}{8}$, $z = \frac{5}{8}$

7. (a) Inconsistent (b) Inconsistent
8. $x_1 = -1$, $x_2 = 4$, $x_3 = 4$

9. $L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{7}{5} & 1 & 0 \\ \frac{3}{5} & \frac{41}{19} & 1 \end{bmatrix}$, $U = \begin{bmatrix} 5 & -2 & 1 \\ 0 & \frac{19}{5} & -\frac{32}{5} \\ 0 & 0 & \frac{327}{19} \end{bmatrix}$

$x = \frac{122}{109}$, $y = \frac{284}{327}$, $z = \frac{46}{327}$

11. $A^{-1} = \begin{bmatrix} 1.2 & -0.4 & 0.2 \\ -0.2 & -0.1 & 0.3 \\ -0.4 & 0.3 & 0.1 \end{bmatrix}$

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**14.** (a) $x = 1.0,\ y = -2.0,\ z = 3.0$
(b) $x_1 = 4.84,\ x_2 = -4.70,\ x_3 = -1.64,\ x_4 = 5.72$

**15.** $x = 1.06,\ y = 1.37,\ z = 1.96$

**16.** (a) Ill-conditioned, (b) Ill-conditioned, (c) Well-conditioned.

**18.** (b) $\lambda_1 = 0,\ \lambda_2 = 3$

$$\begin{bmatrix} -\frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \end{bmatrix}, \begin{bmatrix} \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

**19.** (a) $\lambda = 9$; $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (b) 11.66; $\begin{bmatrix} 0.025 \\ 0.422 \\ 1.0 \end{bmatrix}$

**21.** (a) $\begin{bmatrix} 1 & -5 & 0 \\ -5 & 73/25 & 14/25 \\ 0 & 14/25 & -23/25 \end{bmatrix}$

**24.** $\lambda_1 = 665,\ \lambda_2 = 97$

$$\mathbf{x}_1 = \begin{bmatrix} 0.87 \\ 0.49 \end{bmatrix},\ \mathbf{x}_2 = \begin{bmatrix} -0.49 \\ 0.87 \end{bmatrix}$$

$$A = \begin{bmatrix} 0.76 & -0.65 \\ 0.60 & 0.74 \\ 0.26 & 0.19 \end{bmatrix} \begin{bmatrix} 25.78 & 0 \\ 0 & 9.85 \end{bmatrix} \begin{bmatrix} 0.87 & 0.49 \\ -0.49 & 0.87 \end{bmatrix}$$

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## Chapter 7

**1.** $y(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{8}x^4 + \frac{1}{15}x^5 + \ldots$

$y(0.1) = 1.1053.$

**3.** 4.005, 4.0098 **5.** $y(0.1) = 3.005,\ y(0.2) = 3.020$, etc.

**7.** 1.0202, 1.0408, 1.0619 **8.** 5.0524

**9.** 0.0938, 0.2258 **10.** 1.4983

**12.** 0.0050, 0.0200, 0.0450, 0.0800, 0.1252

**13.** 1.0101, 1.0207, 1.0318, 1.0438



**14.** 1.0569, 1.0713, 1.0871, 1.1048, 1.1244, 1.1464

**15.** 0.0918, 0.1487, 0.2221, 0.3138, 0.4255, 0.5596, 0.7183

**17.** $x(0.1) = 1.1003,\ y(0.1) = 1.1102$

**18.** $v(1.1) = 0.3707,\ \alpha(1.1) = 1.0361$

**19.** 1.0204, 1.0

**20.** With $n = 2$, $y(0.5) = 0.1389$; true value = 0.1505 and with $n = 4$, $y(0.5) = 0.1470$

**22.** Spline solutions: (a) 0.4424 (b) − 2.04
Analytical solutions: (a) 0.4434 (b) − 2.07

## Chapter 8

**1.** (a) $u_1 = 26.66,\ u_2 = 33.33,\ u_3 = 43.33,\ u_4 = 46.66$
(b) $u_1 = 1.57,\ u_2 = 3.71,\ u_3 = 6.57$
$u_4 = 2.06,\ u_5 = 4.69,\ u_6 = 8.06$
$u_7 = 2.00,\ u_8 = 4.92,\ u_9 = 9.00$
(c) $u_1 = 7.8,\ u_2 = 13.6,\ u_3 = 17.8$
$u_4 = 6.6,\ u_5 = 11.9,\ u_6 = 16.2$
$u_7 = 6.6,\ u_8 = 11.2,\ u_9 = 14.3$
(d) $u_1 = 15,\ u_2 = 20,\ u_3 = 25,$
$u_4 = 20,\ u_5 = 20,\ u_6 = 20$
$u_7 = 25,\ u_8 = 20,\ u_9 = 15$

**3.** The values of $f_{ij}$, $\begin{pmatrix} i = 0, 1, 2, 3, 4, 5 \\ j = 0, 1, 2, 3, 4, 5 \\ h = 1, k = \frac{1}{2} \end{pmatrix}$ are:

| $j \backslash i$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0.0 | 24.0 | 84.0 | 144.0 | 144.0 | 0.0 |
| 1 | 0.0 | 42.0 | 78.0 | 78.0 | 57.0 | 0.0 |
| 2 | 0.0 | 39.0 | 60.0 | 67.5 | 39.0 | 0.0 |
| 3 | 0.0 | 30.0 | 53.25 | 49.5 | 33.75 | 0.0 |
| 4 | 0.0 | 26.625 | 39.75 | 43.50 | 24.75 | 0.0 |
| 5 | 0.0 | 19.875 | 35.06 | 32.25 | 21.75 | 0.0 |

**8.** $u_{11} = 0.148,\ u_{21} = 0.053,\ u_{12} = 0.210,\ u_{22} = 0.091$, etc.

Exact solution is $u(x, t) = \sum_{n=1,3\ldots}^{\infty} \frac{8}{\pi^3 n^3} \sin n\pi x \cos n\pi t$

**9.** $u_{11} = 0.1375,\ u_{21} = 0.1075,\ u_{31} = 0.0815$, etc.

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